

Adaptive line enhancer

The present invention relates to adaptive line enhancers and to methods for adaptive line enhancement. Applications for the invention lie in the fields of radar, sonar, communications and other related disciplines where digital signal processing may be required.

BACKGROUND OF THE INVENTION

Detection of sinusoidal signals immersed in noise is a fundamental problem in signal processing. The retrieval of sinusoidal or other narrow-band signals which may have been significantly attenuated, frequency shifted because of Doppler effects and corrupted by interference and noise has conventionally been carried out using analysis of the signal in the frequency domain. This requires the input signal to be Fourier transformed. Once the signal has been Fourier transformed, the strongest spectral component can be detected and a filter designed to either enhance or reject this frequency. For the detection of sinusoids with time-varying frequencies, a Fourier transform with sliding windows can be used. Despite the availability of algorithms such as the fast Fourier transform (FFT) which are computationally efficient when compared to a direct implementation of the discrete Fourier transform, the frequency domain analysis of the input signal is still relatively inefficient when compared to adaptive line enhancement techniques.

Adaptive line enhancement is an alternative technique to frequency domain analysis based on FFTs. It has been shown (B. Widrow and S.D. Stearns, "Adaptive Signal Processing", Prentice-Hall, 1985) that Adaptive Line Enhancers (ALEs) require fewer computations than FFT based techniques and in certain circumstances can be more sensitive detectors of sinusoids. The ALE consists of a filter, and an adaptation rule for changing some feature of the filter's frequency response characteristics. Various combinations of filters and adaptation rules have been proposed, with the most recently reported embodiments comprising a lattice Gray-Markel adaptive notch filter and adaptation rules based on a simplified gradient technique (N.I. Cho, C.-H. Choi and S.U. Lee, "Adaptive Line Enhancement by Using an IIR Lattice Notch Filter," IEEE Trans. Acoust., Speech, Signal Processing, vol. 37, Apr. 1989; P.A. Regalia, "An Improved Lattice-Based Adaptive IIR

Notch filter," IEEE Trans. Signal Processing, vol. 39, pp. 2124-2128, Sept. 1991). It has been shown that such ALEs provide better convergence to the frequency of interest than previous designs and in addition are less sensitive to the finite word length effects which occur in any digital processor.

The transfer function of the Gray-Markel lattice notch filter is expressed as:

$$H_{lattice} = \frac{N(z)}{D(z)} = \left(\frac{1+\alpha}{2} \right) \frac{1+2k_0 z^{-1} + z^{-2}}{1+k_0(1+\alpha)z^{-1} + \alpha z^{-2}} \quad (\text{Equation A})$$

where k_0 determines the notch frequency and where α determines the bandwidth. The notch frequency determining variable k_0 should converge to $-\cos(\omega_0)$ to reject a sinusoid with frequency ω_0 . This filter has zeros on the unit circle at $z_0 = e^{\pm j\omega_0}$, where $\omega_0 = \cos^{-1}(-k_0)$. The -3dB attenuation bandwidth BW of the magnitude response of the Gray-Markel lattice notch filter is determined by the following equation:

$$BW = \cos^{-1} \left(\frac{2\alpha}{1+\alpha^2} \right)$$

A slight gain correction of $\left(\frac{1+\alpha}{2} \right)$ is needed to achieve unity gain in the passband.

The bandwidth and the notch frequency can be controlled separately by changing k_0 and α . This filter structure can easily be implemented using either a direct form realization or a lattice filter structure based on wave digital filters (WDFs) (A. Fettweis and H. Levin and A. Sedlmeyer, "Wave Digital Lattice Filters," Int. J. Circuit Theory Applicat, vol. 2, no. 2, pp. 203-211, June 1974; A. Fettweis, "Wave Digital Lattice Filters: Theory and practice (invited paper)," Proc. IEEE, vol. 74, pp. 270-327, Feb. 1986).

Referring to Fig. 1, there is shown a block diagram showing functional elements for implementing a known wave digital filter realization of the Gray-Markel notch filter response, the transfer function of which is given in Equation A.

In Fig. 1 there is shown an input 110, a first dynamic adapter block 120, a second dynamic adapter block 130, a summing block 140, an amplifier block 150, an output 160, a notch bandwidth determining block 170 and a notch frequency determining block 180.

An input signal 110 is fed to a first input of summing block 140 and to a first input of first dynamic adapter block 120. A first output of first dynamic adapter block 120 is fed to a second input of summing block 140. The output of summing block 140, comprising the result of the addition of input 110 and a first output of first dynamic adapter block 120 is fed to the input of amplifier block 150. Amplifier block 150 has a fixed amplitude gain of 0.5. This gain is achieved by a bit-shift operation and thus does not require a multiplier. The output of amplifier block 150 becomes the output signal 160.

A second output of first dynamic adapter block 120 is left unconnected. A third output of first dynamic adapter block 120 is fed to a first input of second dynamic block 130, said third output in combination with second dynamic adapter 130 forming a feedback path around first dynamic adapter 120.

A first output of second dynamic adapter block 130 is fed back to a second input of first dynamic adapter block 120. The output of notch bandwidth determining block 170 is fed to a third input of first dynamic adapter block 120.

A second output of second dynamic adapter block 130 is left unconnected. A third output of second dynamic adapter block 130 is fed back to a second input of second dynamic adapter block 130. The output of notch frequency determining block 180 is fed to a third input of second dynamic adapter block 130.

Referring to Fig. 2, there is shown a block diagram showing functional elements for implementing the dynamic adapters 120 and 130.

In Fig. 2 there is shown a first input 210, a second input 220, a third input 230, a first subtracter block 240, a multiplier 250, a second subtracter block 260, a third subtracter block 270, a delay block 280, a first output 285, a second output 290 and a third output 295.

The first input 210 is fed to the positive input terminal of first subtracter block 240 and to the negative input terminal of third subtracter block 270. A second input 220 is fed to the negative input terminal of first subtracter block 240 and the negative input terminal of second subtracter block 260. The output of first subtracter block 240, comprising the difference of first input 210 and second input 220 is fed to a first input terminal of multiplier 250. A third input 230 is fed to a second input terminal of multiplier 250. The output of multiplier 250, comprising the product of third input 230 and the output of first subtracter block 240, is fed to the positive input terminal of second subtracter block 260 and to the positive input terminal of third subtracter block 270. The output of second subtracter block 260, comprising the difference of the output of multiplier block 250 and second input 220,

becomes first output 285. The output of third subtracter block 270, comprising the difference of the output of multiplier 260 and first input 210 becomes second output 290 and is fed to delay block 280. Delay block 280 delays the signal by one sample period then feeds it to third output 295.

The time domain response of the dynamic adapter can be evaluated. From an initial state where the values of all inputs and outputs are zero, a train of pulses $a_1, a_2, \dots, a_n, a_{n+1}, \dots$ is applied to first input 210, a train of pulses $b_1, b_2, \dots, b_n, b_{n+1}, \dots$ is applied to second input 220 and the constant K applied to third input 230 then the outputs at the n^{th} time step become:

First output 285 $(a_n - b_n) \times K - b_n$

Second output 290 $(a_n - b_n) \times K - a_n$

Third output 295 $(a_{n-1} - b_{n-1}) \times K - a_{n-1}$

The time domain response of the dynamic adapter shown in feedback configuration in Fig. 5 (and in the dashed box in Fig. 1) can be evaluated. From an initial state where the values of all inputs and outputs are zero, a train of pulses $u(1), u(2), \dots, u(n), u(n+1), \dots$ is applied to the input 510 and the constant K (520) is applied to third input of dynamic adapter block 530, the equation for the output 540 at the n^{th} time step becomes:

Output 540 $y(n) = K \times u(n) + u(n-1) - K \times y(n-1)$

This corresponds to an all-pass transfer function in the z-transform domain of

$$H_{\text{adaptor}} = \frac{K + z^{-1}}{1 + Kz^{-1}}$$

It is desirable to reduce the computational complexity of ALEs.

SUMMARY OF THE INVENTION

It is an object of the present invention to provide an apparatus for adaptive line enhancement using a computationally efficient adaptation mechanism for an adaptive Gray-Markel lattice notch filter.

It is a further object of the present invention to provide a method for adaptive line enhancement using a computationally efficient adaptation mechanism for an adaptive Gray-Markel lattice notch filter.

The inventor has determined that an ALE of reduced computational complexity can be achieved by applying a new adaptation algorithm. An algorithm for adapting the notch frequency determining variable, k_0 , by the recursive Gauss-Newton algorithm with simplified gradient and sgn- μ adaptation rule is as follows:

Algorithm 1:

Input signal is $U(z)$

$$\text{Gray-Markel notch filter: } H_{\text{lattice}} = \frac{N(z)}{D(z)} = \left(\frac{1+\alpha}{2} \right) \frac{1+2k_0 z^{-1} + z^{-2}}{1+k_0(1+\alpha)z^{-1} + \alpha z^{-2}}$$

$$\text{Output signal is } Y(z) = H_{\text{lattice}} \times U(z)$$

Attempt to minimize the expected value of the energy of the output $E(Y^2)$

Filter Operations (for each input sample):

Calculate notch filter output $y(n)$ using WDF filter structure (see Fig. 1)

Adapt k according to Equation C (below) in which $x(n-1)$ is the simplified gradient of the current sample and relates to input $u(n)$ as shown in Equation B.

$$x(n) = \frac{1}{D(z)} u(n) = u(n) - k(n)(1+\alpha)x(n-1) - \alpha x(n-2) \quad (\text{Equation B})$$

$$k(n+1) = k(n) - \text{sgn}[x(n-1)y(n)]\mu \quad (\text{Equation C})$$

μ is the adaptation constant.

Stability monitoring: clip $k(n+1)$ within range $]-1 \ 1[$

The adaptation constant μ determines the rate of convergence of the algorithm on k_0 , and also puts bounds upon the achievable mean accuracy of the estimation of k_0 .

To avoid the calculation of the simplified gradient $x(n-1)$ needed for the update of k_0 , the internal variables of the WDF filter structure of the Gray-Markel notch filter are studied.

The table below shows a z-transform domain transfer function of the internal variables in the signal flow graph of the wave digital filter structure shown in Fig. 1.

<i>Internal Variable</i>	<i>Transfer function</i>
Input	1
Output	$\left(\frac{1+\alpha}{2}\right) \frac{1+2k_0z^{-1}+z^{-2}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$
Out1 Adapter 1	$\left(\frac{1+\alpha}{2}\right) \frac{\alpha+k_0(1+\alpha)z^{-1}+z^{-2}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$
Out3 Adapter 1 = In1 Adapter 2	$(\alpha-1)z^{-1} \frac{1+k_0z^{-1}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$
In2 Adapter 1 = Out1 Adapter 2	$(\alpha-1)z^{-1} \frac{k_0+z^{-1}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$
In2 Adaptor2 = Out3 Adapter 2	$\frac{(\alpha-1)(k_0-1)z^{-2}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$
Out2 Adapter 2	$\frac{(\alpha-1)(k_0-1)z^{-1}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$

It can be seen from the definition of simplified gradient $x(n-1)$ given in Equation B and from the z-transform domain transfer functions of the internal variables of the wave digital filter shown in Fig. 1 that the third output of second dynamic adapter 130 corresponds to $(\alpha-1)(k_0-1)x(n-2)$ and that the second output of second dynamic adapter 130 corresponds to $(\alpha-1)(k_0-1)x(n-1)$.

Since both $|\alpha|<1$ and $|k|<1$ as a fundamental requirement for stability of the Gray-Markel filter, the product $(\alpha-1)(k_0-1)$ is always positive. Therefore:

$$\text{sgn}[x(n-1)y(n)] = \text{sgn}[y(n)] \text{sgn}[\text{Out}_2\text{Adaptor}_2] \quad (\text{Equation D})$$

This equation makes the calculation of simplified gradients unnecessary and leads to a new algorithm for an ALE using a low complexity adaptive lattice notch filter based on wave digital filters. The algorithm is as follows:

Algorithm for Adaptive Line Enhancer using a Low Complexity Adaptive Lattice Notch Filter based on Wave Digital Filters:

Input signal is $u(n)$ with n starting at time 0 ($U(z)$ in frequency domain notation)

Gray-Markel notch filter: $H_{lattice} = \frac{N(z)}{D(z)} = \left(\frac{1+\alpha}{2}\right) \frac{1+2k_0z^{-1}+z^{-2}}{1+k_0(1+\alpha)z^{-1}+\alpha z^{-2}}$

Output signal is $Y(z) = H_{lattice} \times U(z)$

5

Attempt to minimize the expected value of the energy of the output $E(Y^2)$

Initialization:

Initialize $Out_3Adaptor_1(-1)$

10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867
868
869
870
871
872
873
874
875
876
877
878
879
880
881
882
883
884
885
886
887
888
889
890
891
892
893
894
895
896
897
898
899
900
901
902
903
904
905
906
907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000

Initialize $Out_3Adaptor_2(-1)$

Filter Operations (for each input sample $u(n)$):

Calculate notch filter output $y(n)$ using WDF filter structure with input $u(n)$ (see Fig. 1):

$$Out_3Adaptor_1(n) = Out_2Adaptor_1(n-1)$$

$$Out_3Adaptor_2(n) = Out_2Adaptor_2(n-1)$$

$$Out_1Adaptor_2(n) = k(n) \times [Out_3Adaptor_1(n) - Out_3Adaptor_2(n)] - Out_3Adaptor_2(n)$$

$$Out_2Adaptor_2(n) = k(n) \times [Out_3Adaptor_1(n) - Out_3Adaptor_2(n)] - Out_3Adaptor_1(n)$$

$$Out_1Adaptor_1(n) = \alpha \times [u(n) - Out_1Adaptor_2(n)] - Out_1Adaptor_2(n)$$

$$Out_2Adaptor_1(n) = \alpha \times [u(n) - Out_1Adaptor_2(n)] - u(n)$$

$$y(n) = 0.5 \times [u(n) + Out_1Adaptor_1(n)] - u(n)$$

Update of the variable k determining the notch frequency (see Fig. 3):

$$k(n+1) = k(n) - \text{sgn}[y(n)]\text{sgn}[Out_2Adaptor_2] \times \mu$$

μ is the adaptation constant.

Thus $Out_2Adaptor_2$ can be used as an update function (UPDATEFN) for an update of the variable k used to determine the notch frequency in a given sampling period. As referred to

above, UPDATEFN therefore has a transfer function in the z-transform domain, in this case in an n^{th} sampling period of:

$$\frac{(\alpha - 1)(k(n) - 1)z^{-1}}{1 + k(n)(1 + \alpha)z^{-1} + \alpha z^{-2}}$$

5

Stability monitoring: clip $k(n+1)$ within range $]-1 \ 1[$.

According to the present invention in a first aspect, there is provided an adaptive line enhancer comprising an adaptive Gray-Markel lattice notch filter having an adaptive notch frequency, in which the notch frequency is determined according to a notch frequency variable k , characterized in that the value of k for the $n+1^{\text{th}}$ sample period is determined according to the following equation:

$$k(n+1) = k(n) - \text{sgn}[y(n)]\text{sgn}[\text{UPDATEFN}] \times \mu$$

15

in which $y(n)$ is the notch filter output, μ is the adaptation constant, and UPDATEFN has a transfer function in the z-transform domain of:

$$\frac{(\alpha - 1)(k(n) - 1)z^{-1}}{1 + k(n)(1 + \alpha)z^{-1} + \alpha z^{-2}}$$

20

in which α determines the bandwidth and $k(n)$ is a variable for determining the current notch frequency.

According to the present invention in a second aspect, there is provided a method for adaptive line enhancement, comprising adaptive line enhancing an adaptive Gray-Markel lattice notch filter with an adaptive notch frequency, in which the notch frequency is determined according to a notch frequency variable k , characterized in that the value of k for the $n+1^{\text{th}}$ sample period is determined according to the following equation:

$$k(n+1) = k(n) - \text{sgn}[y(n)]\text{sgn}[\text{UPDATEFN}] \times \mu$$

30

in which $y(n)$ is the notch filter output, μ is the adaptation constant, and $UPDATEFN$ has a transfer function in the z -transform domain of:

$$\frac{(\alpha - 1)(k(n) - 1)z^{-1}}{1 + k(n)(1 + \alpha)z^{-1} + \alpha z^{-2}}$$

in which α determines the bandwidth and $k(n)$ determines the current notch frequency.

This algorithm for adapting the notch frequency enables $UPDATEFN$ and the notch frequency variable to be directly calculated from knowledge of internal variables of the wave digital filter

The update rule for $k(n)$ in this algorithm is very simple and requires, when compared with the prior art notch filter of Fig. 1, only an extra computational load of one addition, two sign operators and an EXOR operator (for doing the multiplication of the two signs). Together with the operation for calculating the WDF filter structure this makes a total of two multiplications, eight additions, one bit shift, two sign operators and one EXOR operator for each processed input sample.

This represents a saving of three multiplications and two additions which would otherwise be needed to calculate the simplified gradient each sample period, according to Equation B.

Claims 2-5 define advantageous apparatus for putting the present invention in to effect.

Claims 7-10 define advantageous ways in which the method of the present invention may be implemented.

BRIEF DESCRIPTION OF THE DRAWINGS

For a better understanding of the invention, and to show how an embodiment of the same may be carried into effect, reference will now be made, by way of example, to the accompanying diagrammatic drawings in which:

Fig. 1 is a schematic drawing illustrating means of implementing a prior art Gray-Markel notch filter using a wave digital filter;

Fig. 2 shows a schematic drawing illustrating means of implementing the "dynamic adapter" shown in Fig. 1;

Fig. 3 shows a schematic drawing illustrating an adaptive line enhancer according to the present invention;

Fig. 4 shows the results of a frequency hop experiment used to evaluate the effectiveness of an embodiment of the present invention; and

Fig. 5 shows a schematic drawing illustrating the "dynamic adapter" in a feedback configuration.

DESCRIPTION OF PREFERRED ENBODIMENTS

Referring now to Fig. 3, there is shown a block diagram that is identical to the Fig. 1 schematic except that it includes additional feedback elements that implement the low complexity adaptation algorithm that forms the basis of the invention.

In Fig. 3 there is shown an input 305 $u(n)$, a first dynamic adapter block 310, a second dynamic adapter block 315, a first summing block 320, an amplifier block 325, an output 330, a notch bandwidth determining block 335, a notch frequency determining block 340, a first signum function block 345, a second signum function block 350, a first multiplier 355, a second multiplier 360, an adaptation speed determining block 365, a second summing block 370, an amplitude limiting block 375 and a delay block 380.

An input signal 305 is fed through a first input of first summing block 320 and to a first input of first dynamic adapter block 310. A first output of first dynamic adapter block 310 is fed to a second input of first summing block 320. The output of first summing block 320, comprising the result of the addition of input 305 and a first output of first dynamic adapter block 310 is fed to be input of amplifier block 325. Amplifier block 325 has a fixed amplitude gain of 0.5. This gain is achieved by a bit-shift operation and thus does not require a multiplier. The output of amplifier block 325 becomes the output signal 330 and is also fed to the input of second signum function block 350. A second output of first dynamic adapter block 310 is left unconnected. A third output of first dynamic adapter block 310 is fed to a first input of second dynamic adapter block 315. A first output of second dynamic adapter block 315 is fed back to a second input of first dynamic adapter block 310. The output of notch bandwidth determining block 335 is fed to a third input of first dynamic adapter block 310. A second output (Out2) of second dynamic adapter block 315 is fed to the input of first signum function block 345. A third output of second dynamic adapter block 315 is fed back to a second input of second dynamic adapter block 315. The output of first signum function block 345 is fed to a first input of first multiplier 355, and the output of second signum function block 350 is fed to a second input of first multiplier 355. The output

of first multiplier 355, comprising the product of the output of first signum function block 345 and second signum function block 350 is fed to a first input of second multiplier 360. The output of an adaptation speed block 365 is fed to a second input of second multiplier 360. The output of second multiplier 360, comprising the product of the output of the first multiplier block 355 and the output of adaptation speed block 365 is fed to a first input of second summing block 370. The output of notch frequency determining block 340 is fed to a third input of second dynamic adapter block 315 and to a second input of second summing block 370. The output of second summing block 370, comprising the sum of the output of second multiplier 360 and the output of notch frequency determining block 340 is fed to the input of amplitude (saturation) limiting block 375. The output of amplitude limiting block 375 is fed to the input of delay block 380. The output of delay block 380 becomes the updated value of notch frequency determining block 340 and accordingly is fed to a third input of second dynamic adapter block 315.

Amplitude limiting block 375 prevents $k(n+1)$ from becoming ≥ 1 or ≤ -1 .

When $|k(n+1)| \geq 1$ the notch filter becomes unstable. To prevent instability $k(n+1)$ is clipped in to the open interval $]-1, 1[$. This is done as follows:

If $k(n+1) \geq \text{ClipValue}$ then $k(n+1) = \text{clipvalue}$

If $k(n+1) \leq -\text{ClipValue}$ then $k(n+1) = -\text{clipvalue}$

With clipvalue being slightly less than 1, e.g. 0.999. This is also referred to as stability monitoring.

The second output Out 2 of second dynamic adapter 315 is used to generate the $k(n+1)$ value used as a variable to determine the update for the adaptive coefficient determining the notch frequency. The signum of Output2 of second dynamic adapter 315 is generated by first signum block 345, which is multiplied by first multiplier 355 with the signum of the output of amplifier block 325 (which is $y(n)$, the notch filter output), the signum of the output of amplifier block 325 being carried out by second signum block 350. This therefore generates $\text{sgn}[y(n)]\text{sgn}[\text{Out2}]$ as the output of first multiplier 355. This is then multiplied by adaptation constant μ from adaptation speed block 365 at second multiplier 360 and subtracted from the current $k(n)$ to generate $k(n+1)$.

Thus the second output Out2 of second dynamic adapter 315 is used as an update function (UPDATEFN). As shown in the table above (reference Output₂Adaptor₂) UPDATEFN has a transfer function in the z-transfer domain, for an n^{th} sample of:

$$\frac{(\alpha - 1)(k(n) - 1)z^{-1}}{1 + k(n)(1 + \alpha)z^{-1} + \alpha z^{-2}}$$

The embodiment of the present invention represented diagrammatically by the block diagram shown in Fig. 3 has significant advantages over prior realizations of ALEs, particularly in terms of minimizing the amount of hardware needed to carry out the ALE procedure and minimizing the computational load needed to carry out the ALE procedure on any digital processor. Fig. 4 shows the results of a frequency hop experiment in which the input signal supplied to the embodiment of the invention shown in Fig. 3 is a sine wave immersed in white noise and sampled at a sampling rate f_s equal to 16kHz. The frequency of the sine wave changes randomly every 1000 samples. The first graph (from top to bottom) shows the desired frequency (des freq) with $\alpha=0.8$, $\mu=0.005$ and $\text{SNR}=23\text{dB}$. The second graph shows the estimated frequency (est freq) using an ALE of the described embodiment of the present invention. The third graph corresponds to the first graph except that $\alpha=0.7$, $\mu=0.001$ and $\text{SNR}=4.9\text{dB}$. The fourth graph shows the corresponding estimated frequency using an ALE of the described embodiment of the present invention.

The embodiment of the present invention is used to make an estimate of this desired frequency $f_{\text{freq. estim.}}(n)$ for each time step by using:

$$f_{\text{freq. estim.}}(n) = \frac{f_s}{2\pi} \cos^{-1}[-k(n)] \quad (\text{Equation E})$$

To quantify the achievable accuracy of the frequency estimation given by this algorithm in case of the frequency hop experiment, it can be proved that the standard deviation of the estimated frequency $\omega_0 = \cos^{-1}(-k_0)$ shows following proportional relationship:

$$\sigma_{\text{freq. estim.}} \approx \frac{\mu}{\sqrt{1 - k^2_{\text{real}}}} \quad (\text{Equation F})$$

The estimate of the desired frequency will be converged on to the desired frequency in a time showing the following proportional relationship:

$$T_{\text{conv}} \approx \frac{1}{\mu} \text{ samples} \quad (\text{Equation G})$$

By choosing appropriate values for α and μ by inspection of equations F and G and/or basic experimentation, useful adaptive sinusoid tracking can be achieved for various signal to noise ratios as is demonstrated in Fig. 4.

Accordingly the variable k used for determining the notch frequency is itself determined by an output of the second dynamic adapter i.e. internally rather than externally.

All of the features disclosed in this specification (including any accompanying claims, abstract and drawings), and/or all of the steps of any method or process so disclosed, may be combined in any combination, except combinations where at least some of such features and/or steps are mutually exclusive.

Each feature disclosed in this specification (including any accompanying claims, abstract and drawings), may be replaced by alternative features serving the same, equivalent or similar purpose, unless expressly stated otherwise. Thus, unless expressly stated otherwise, each feature disclosed is one example only of a generic series of equivalent or similar features.

The invention is not restricted to the details of the foregoing embodiment(s). The invention extend to any novel one, or any novel combination, of the features disclosed in this specification (including any accompanying claims, abstract and drawings), or to any novel one, or any novel combination, of the steps of any method or process so disclosed.